Swami Ramanand Tirth Marathwada University, Nanded, Maharashtra, India

(UGC recognized NAAC Re-accredited B-grade State University)

"Some Studies in Spline Curve Applications for Modeling and Solving Engineering Systems governed by Differential Equations"

Pre-Synopsis submitted
For the confirmation of the Ph.D. Topic

In the subject
Mathematics faculty of Science & Technology

Submitted by

Mr. Santosh Vijaykumar Chavan
Research Scholar
Dayanand Science College, Latur
Latur – 413531, Maharashtra State, India
Email id: kvmsantosh@gmail.com, Mobile No. 8626001133

Under the Supervision of

Dr. Sidheshwar S. Bellale
Department of Mathematics
Dayanand Science College, Latur
Latur – 413531, Maharashtra State, India
Email id: sidhesh.bellale@gmail.com, Mobile No. 9405417417

Research Centre
Department of Mathematics
Dayanand Science College, Latur
Latur – 413531, Maharashtra State, India
Phone: 9405417417

1] Introduction:

Spline curves are the mathematical constructions used to represent smooth and flexible curves through a number of control points, known as knots. Spline curves are actually piecewise-defined polynomial functions joined to each other such that smoothness and continuity across their segments between the control point is maintained. Splines are defined using specific parameters and conditions that determine their shape and behavior, such as degree, knots, and continuity constraints.

Spline curves are of different forms, each having specific properties and applications. The most commonly used types of spline curves are Linear Splines, Quadratic Splines, Cubic Splines, B-Splines (Basis Splines), NURBS (Non-Uniform Rational B-Splines), Hermite Splines, Bezier Splines, Cardinal Splines etc.

A linear spline is the simplest type of spline, consisting of straight-line segments that connect a series of control points i.e. knots. A quadratic spline is a piecewise-defined curve where each segment is polynomial of degree two. A cubic spline is a piecewise polynomial of degree three, that is continuous up to the second derivative at the control points. B-splines are a generalization of spline curves where the curve is represented as a linear combination of basis functions defined by a knot vector. NURBS are an extension of B-splines, incorporating weights and rational functions which ensures flexibility in shape representation. Hermite splines are defined by their endpoints and associated tangents at the corresponding point, rather than intermediate control points. A Bezier spline is defined using control points, with the curve following a parametric representation. It is a special type of spline that passes through a set of control points, using a tension parameter to adjust the tightness of the curve.

Spline curves are used as an important tool in computational techniques and mathematical modeling, known for their flexibility and accuracy in representing complex shapes, stuctures and functions. Their flexibility and accuracy has made them an important key in fields such as Computer-Aided Design (CAD), computer graphics, engineering simulations, and data interpolation. Being spline curve has the properties like adaptability, smoothness, and computational efficiency, spline curves play an invaluable role in both theoretical mathematics and applied engineering, forming the foundation for advanced technological applications.

The motivation for this research lies in the large number of applications of spline curves across different domains of science and engineering. Now a days, with advancements in technology, the demand for accurate and computationally efficient methods to model and analyze real-world problems has increased. Since spline curves have the ability to approximate and interpolate data effectively, it has become a cornerstone in addressing these challenges. For instance, in CAD, spline curves are widely used to design complex structures with accuracy, while in graphics, they provide smooth animations and realistic visual effects. In engineering simulations, spline-based techniques enable accurate representation of physical phenomena.

This work aims to connect theoretical concepts of spline curves with practical engineering applications, making the study both useful in academics and valuable in real-world use. Moreover, this work aims to address certain problems related to differential equations arising in different branches of engineering like mechanical, civil etc. By conducting this study, the aim is to not only contribute to the existing knowledge of spline curves but also provide innovative solutions that can be implemented in real-world scenarios.

2] Literature Review:

Historical Developments in Spline Curves

The study of spline curves has its origins in the mid-20th century, driven by the need for smooth and flexible curve representations in numerical analysis and engineering. Early developments in spline theory were heavily influenced by mechanical drawing tools used in shipbuilding and aeronautics. These tools, such as drafting splines, inspired the mathematical formulation of splines to approximate and interpolate curves.

Major breakthrough occurred when A. Schoenberg introduced the concept of spline functions in 1946. Schoenberg's work on piecewise polynomial functions provided a formal mathematical foundation for splines, laying the groundwork for their theoretical development. During the 1950s, splines were adopted in engineering disciplines for tasks like structural analysis and curve fitting. The introduction of cubic splines, characterized by their second-derivative continuity, marked a significant advancement, making splines a preferred choice for data interpolation. In 1960s, the development of B-splines by Carl de Boor and M.J.D. Powell, along with the independent work of Cox, provided a generalized framework

for spline representation. B-splines allowed for greater flexibility and local control, revolutionizing computer-aided design (CAD) and graphics. During the 1970s, Non-Uniform Rational B-Splines (NURBS) were introduced as an extension of B-splines to handle more complex shapes, such as circles and free-form surfaces. NURBS became a standard in CAD and 3D modeling due to their ability to represent both simple and complex geometries accurately.

Recent Developments in Spline Curves

In the 1960s, researchers began applying splines to solve differential equations. The spline collocation method emerged, where splines approximated solutions by satisfying differential equations at discrete collocation points. This method was particularly effective for boundary value problems (BVPs). The book, *The Theory of Splines and Their Applications* by Ahlberg, Nilson, and Walsh (1967) was one of the earliest comprehensive texts on splines and their use in numerical computations, including differential equations. During the 1980s, splines were extensively used to solve higher-order ODEs, such as those arising in beam theory and elasticity. Cubic and quartic splines became popular due to their ability to ensure smoothness of higher derivatives. Researchers extended spline methods to PDEs, particularly for problems in engineering and physics. Sweatman (1995) developed spline collocation methods for nonlinear boundary value problems. Ciarlet (1990s) integrated splines with finite element methods for solving PDEs in elasticity and fluid mechanics. In the 2000s, splines were integrated into finite element methods (FEM), enabling the solution of PDEs in multi-dimensional engineering applications. Splines began to be used not only for solving differential equations but also for optimizing engineering systems governed by these equations. Hughes (2005) introduced iso-geometric analysis, revolutionizing the use of splines in solving PDEs. Li and Xu (2010) worked on spline-based numerical methods for PDEs, focusing on heat conduction and fluid flow problems.

Splines are now widely used in structural analysis, fluid dynamics, and thermal engineering to solve ODEs and PDEs. For example, they are employed in vibration analysis, stress distribution, and flow simulations. In the present study, we are interested in modeling and solving boundary value problems and initial value problems and to compare the outcomes with the results obtained by finite element method, finite difference method, numerical methods etc.

Work Carried out at National & International level

In the research paper entitled "B-Spline Collocation method for numerical solution of non-linear Kawahara and Modified Kawahara equations", Amir S. B. [1] has applied a collocation method for solving the Kawahara and modified Kawahara equations. Idris D., Dursan I. [2] used Cubic B-spline collocation method to the time-splitted Burgers' equation. Quadratic B-spline collocation method is used to get numerical solution of the space-splitted Burgers' equation.

In the paper entitled "Cubic B-Spline Collocation Method for Sixth Order Boundary Value Problems", Y. Showri Raju [7], developed a collocation method with cubic B-splines as basis functions for getting the numerical solution of a general linear sixth order boundary value problem. Soumia Tayebi & Shaher Momani [6], in their paper "The cubic B-spline interpolation method for numerical point solutions of conformable boundary value problems" employed the cubic B-spline method for constructing the approximate solutions of a class of fractional two-point boundary value problems.

Joan Goh, Ahmad Abd. Majid, and Ahmad Izani Md. Ismail [8] have obtained numerical solutions of one-dimensional heat and advection-diffusion equations by collocation method based on cubic B-spline in their paper titled "Cubic B-Spline Collocation Method for One-Dimensional Heat and Advection-Diffusion Equations". In the paper entitled "Cubic B-splines collocation method for a class of partial integro-differential equation", M. Gholamian, J. Saberi-Nadjafi [4] a numerical method is proposed to estimate the solution of initial-boundary value problems for a class of partial integro-differential equations.

Now, in this research work our aim to solve different differential equations i.e. ordinary and partial differential equations arising in engineering field, by using different techniques of spline curve. For example, will try to solve equations like Partial differential equation of Heat Conduction $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$, the Fourth order ordinary differential equation of Beam Deflection $EI \frac{d^4 y}{dx^4} = q(x)$, Second order ordinary differential equation of Fluid flow in Porous media $k \frac{d^2 h}{dx^2} = 0$, Ordinary differential equations of Chemical Kinetics $\frac{dC}{dt} = -kC$, Heat Transfer in Electrical components $\frac{\partial T}{\partial t} = k\nabla^2 T$, Hyperbolic Partial Differential equation i.e. Wave Equation of Seismic analysis $\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$ etc.

3] **Research Objectives:**

The General and specific research objectives of this study are as follows:

General Objective:

1) To develop practical applications of spline curve techniques to solve critical engineering problems across multiple branches of engineering.

2) To provide innovative solutions that enhance the adaptability and computational efficiency of spline-based methods.

3) To publish findings in research papers that contribute to real-word problems.

Specific Objectives:

1) To apply spline-based techniques to solve ordinary differential equations arising in mechanical engineering domains like fluid dynamics, heat transfer, one dimensional and two dimensional heat flow etc.

2) To develop mathematical modeling of different curves, shapes arising in Civil Engineering like road curves, bridges etc. using spline curves.

3) To apply spline-based techniques to solve partial differential equations arising in civil engineering domains like bending of beam, stress-strain analysis etc.

4] Research Methodology:

The methodology highlights the steps and approaches used to study and develop spline-based solutions for differential equations, both ordinary (ODEs) and partial (PDEs), arising in engineering systems. It will include mathematical formulation, computational techniques, and applications.

1) Conduct an extensive review of existing literature on spline curves, including polynomial splines, B-splines, and NURBS (Non-Uniform Rational B-Splines). Identify gaps in the current knowledge and unexplored areas in the application of spline curves. Formulate precise research questions and hypotheses based on these gaps.

2) Initially, we will study and short list different spline-based numerical methods for solving ordinary and partial differential equations. Then will try to analyze the accuracy and efficiency of these methods compared to traditional approaches to demonstrate the applicability of spline methods in real-world engineering problems.

3) Will try to construct spline functions (linear, quadratic, cubic, and higher-order splines) with continuous derivatives at knots. Investigate and compare different types of splines: Polynomial Splines, B-splines (Basis Splines), Non-Uniform Rational B-Splines (NURBS). Formulate basis functions suitable for higher-order differential equations.

4) Explore collocation methods, Galerkin methods and finite element methods (FEM) with splines. Derive error bounds for spline-based solutions. Study the convergence properties and order of accuracy. Use programming languages like MATLAB, Python, or C++ for implementing spline algorithms.

5) Choose differential equations from various branches of engineering, such as: Beam deflection in mechanical engineering, Heat transfer in civil and chemical engineering, Fluid flow in aerospace engineering, Electromagnetic wave propagation in electrical engineering etc. Conduct case studies on real-world problems to test the practical applicability of spline methods.

6) Compare results with experimental or industrial data wherever available. Evaluate the performance of spline methods based on Computational efficiency, Solution accuracy etc. Compare the spline-based solutions with traditional numerical methods like finite difference and finite element methods in terms of computational efficiency and accuracy. Use metrics such as Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and relative error for quantitative evaluation.

7) Expected Outcomes

The expected outcomes of this research include: (i) Development of alternative methods of finding solutions of differential equations in engineering problems. (ii) Implementation of computational methods to provide efficient and accurate solutions. (iii) Practical applications and case studies that demonstrate the value of Spline curves in real-world engineering practice.

5] Importance of study:

The study holds significant importance in both theoretical and practical domains. Spline curves, with their valuable properties of flexibility and smoothness, offer powerful tools for solving complex problems in engineering.

1) Spline curves are powerful tools and can be efficiently used for solving differential equations, particularly when traditional analytical methods have limitations. Since spline curve have the ability to approximate functions with smoothness and precision makes them valuable in numerical solutions of ordinary differential equations (ODEs) and partial differential equations (PDEs).

2) Traditional numerical methods for solving ordinary and partial differential equations (ODEs and PDEs), such as finite difference, finite element and Runge-Kutta methods, often face challenges related to computational efficiency, handling irregular geometries or ensuring smooth solutions. Spline curves, with their smoothness and flexibility, provide an alternative approach handling these challenges more effectively.

3) Differential equations play an important role in modeling physical phenomena in various engineering fields such as structural analysis, fluid dynamics, heat transfer, and electromagnetic wave propagation. By applying spline methods, we can obtain highly accurate and smooth solutions, particularly for complex boundary conditions and non-linear problems.

4) The versatility of spline-based methods makes them applicable across multiple branches of engineering: Mechanical Engineering: Analysis of beam deflection, vibrations, and dynamic systems. Civil Engineering: Structural stability, groundwater flow, and heat conduction problems. Chemical Engineering: Reaction kinetics and diffusion processes. Aerospace Engineering: Compressible fluid flow and heat transfer in re-entry vehicles. Electrical Engineering: Electromagnetic wave propagation and thermal analysis of components.

5) Spline theory is a foundational tool in computational mathematics. This research contributes to its theoretical development by exploring new formulations of spline functions for differential equations. This will Expand the range of engineering problems that can be solved using splines.

6) This study addresses a critical need in both numerical analysis and engineering applications by exploring the importance of spline curves as an effective tool for solving differential equations. Its outcomes will also provide practical solutions to engineering challenges

6] The tentative year wise plan for proposed research work:

Year 01: Will study the basic literature by collecting the research papers, books etc. on current applications of spline curve and identify research gaps. Will attend conference, workshop and seminars in relevant topic and collect research papers for study and further research work. I will attend course like NPTEL, SWAYAM etc. to update the content. I will visit different research centers, universities, institutions, libraries, internet etc.

Year 02: Will try to identify research gaps. Select key problems from real-word and engineering field, define boundaries, and formulate initial mathematical models. Begin solving simplified models analytically; explore solution methods implementing spline curve methods for more complex problems of differential equations. Review progress and refine research direction based on feedback. Begin writing research papers.

Year 03: Will study some applications of spline curve teqniques. Investigate new mathematical insights and will publish research papers in reputed journals. Review progress and refine research direction based on feedback by the supervisor. Will begin drafting thesis chapters. Final thesis writing, review and preparation for defense.

7] References:

1) B-Spline Collocation method for numerical solution of non-linear Kawahara and Modified Kawahara equations, Amir Saboor Bagedzadeh, TWMS J. App. Eng. Math. V.7, N.2, 2017, pp. 188-199

2) B-Spline Collocation Methods for Numerical Solutions of the Burger's Equation, Idris Dag, Dursun Irk and Alis Ahin, https://www.researchgate.net/publication/26411618

3) B-Spline Collocation Method for Solving Singularly Perturbed Boundary Value Problems, Bin Lin, Journal of Applied Mathematics and Physics, 2016, 4, 1699-1704

4) Cubic B-splines collocation method for a class of partial integro-differential equation, M. Gholamian, J. Saberi-Nadjafi, Alexandria Engineering Journal (2018) 57, 2157–2165

5) Piecewise Regression Using Cubic Spline-A Case Study, Sarita Gajbhiye Meshram and P. L. Powar, International Journal of Hybrid Information Technology Vol. 10, No.1 (2017), pp. 75-84

6) The cubic B-spline interpolation method for numerical point solutions of conformable boundary value problems, Soumia Tayebi, Shaher Momani, Alexandria Engineering Journal (2022) 61, 1519–1528

7) Cubic B-Spline Collocation Method for Sixth Order Boundary Value Problems, Y. Showri Raju, International Journal of Scientific and Innovative Mathematical Research (IJSIMR) *Volume 5, Issue 7, 2017, PP 1-13*

8) Cubic B-Spline Collocation Method for One-Dimensional Heat and Advection-Diffusion Equations, Joan Goh, Ahmad Abd. Majid, and Ahmad Izani Md. Ismail, Hindawi Publishing Corporation, Journal of Applied Mathematics Volume 2012, Article ID 458701, 8 pages doi:10.1155/2012/458701

10) One Dimensional Heat Equation, by Ravi Agarwal, https://www.academia.edu/71965163/The_One_Dimensional_Heat_Equation?auto=download

11) Numerical Simulation of one dimensional Heat Equation: B-Spline Finite Element Method by V. Dabral, S. Kapoor, S. Dhawan

12) Numerical solution of the heat equation with nonlocal boundary conditions, Liu, Y., J. Comput. Appl. Math. 110, 115– 127 (1999).

13) The Solution of One-Dimension of Heat Conduction Equation by Adomian Decomposition Method, A. Sadeghinia, P. Kumar, International Journal of Physical, Chemical & Mathematical Sciences, Vol. 2; No. 2: ISSN: 2278-683X (July-Dec. 2013)

14) Application of Curve Fitting in Indian Structures by Sunita Daniel, Abin Sam, Journal of Civil Engineering and Environmental Technology Print ISSN: 2349-8404; Online ISSN: 2349-879X; Volume 1, Number 3; August, 2014 pp. 109-113

Mr. Santosh V. Chavan
Research Scholar
Dayanand Science College, Latur

Dr. Sidheshwar S. Bellale
Research Supervisor
Dayanand Science College, Latur